Alternative Mechanisms of Peer Achievement Spillovers: Implications for Identification and Policy*

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Abstract

This paper provides a link between the theoretical and empirical models of peer spillovers in the education context, informing the interpretation and identification of the parameters of the empirical model and their applicability to policy. The point of departure is to consider why peer achievement spillovers in educational production. I compare two theories, that peer achievement proxies for unobserved peer ability (a predetermined characteristic) or effort (a simultaneous choice). While the literature does not tend to distinguish between these mechanisms, I show that the implications for identification and policy are quite different and that the tendency to focus only on spillovers from predetermined characteristics of students may be misguided. The theoretical model also helps to clarify the interpretation of spillovers from peer characteristics. I show that the marginal effect of peer characteristics may actually differ in sign depending on the underlying mechanism of peer influence, potentially helping to reconcile some of the mixed evidence regarding contextual effects in the literature. Finally, I discuss the relevance of these results for policy, particularly focusing on optimal grouping.

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1 Introduction

Both theoretical models and empirical evidence of peer effects abound, but largely these literatures have developed in isolation.[1] Theory suggests a number of mechanisms through which peers can affect outcomes, for instance, in setting norms of behavior, affecting expectations, or generating direct externalities. Statistical models generally distinguish between two types of peer effects–*endogenous* effects deriving through peer behavior and *contextual* or *exogenous* effects deriving through peer characteristics. The theoretical models are often used only to motivate the existence of peer spillovers, but not connected structurally to the two types of peer effects estimated in the empirical model. In the education context, this has partially contributed to a lack of consensus in the conceptualization of endogenous peer spillovers and the mixed evidence regarding contextual effects.[2] In this paper, I show that directly linking the statistical model to underlying theoretical models of endogenous peer spillovers in the achievement context informs the interpretation and identification of the parameters of the empirical model and also helps clarify what parameters are relevant to policy.

I take as my starting point the question—why does peer achievement belong in the production function? As emphasized by Hanushek et al. (2003) and others, this is not immediately evident. Educational output is generally measured as performance on annual standardized exams, which, in the absence of cheating, is not a group effort. In particular, I consider two existing hypotheses—that peers affect achievement (1) through unobserved predetermined characteristics, such as ability, and (2) through unobserved behaviors, such as effort or motivation. Both theories share in common that peer achievement in reality is proxying for an unobservable. This suggests that conditioning on a given level of peer achievement in the achievement production function can bias estimates of the contextual effect toward zero. For a given level of peer achievement, "better" peer characteristics actually predict a lower level of peer effort or ability. Thus, estimates of the contextual effect pick up the direct effect of peer characteristics net of an indirect effect deriving through the effort (or ability) channel. In the extreme, the resulting peer spillover estimates may contradict intuition suggesting that if individual characteristics, such as parental education, are positively correlated with achievement, then having more peers with those characteristics will positively (or at least non-negatively) affect achievement. In the more mild cases, the

[1] See Manski (2000) and Brock and Durlauf (2001*b*) for discussion.

[2] For instance, see Schofield (1995) for a review of the mixed evidence regarding the effect of desegregation.

opposing channels of direct and indirect spillovers from peer characteristics may lead us to understate the exogenous effects or erroneously conclude that exogenous effects do not exist.

While both the ability or effort mechanisms of peer achievement spillovers share in common the prediction that estimates of a direct contextual effect in the econometric model will be biased, contrasting the two mechanisms has important implications for identification. The reflection problem, as first described by Manski (1993), highlights two challenges in the identification of peer effects—(1) separating endogenous from exogenous peer effects and (2) separating a *social effect*, the combination of endogenous and exogenous peer effects, from other unobserved correlated effects, such as teacher quality, that lead to similar outcomes among peers. The literature on peer spillovers in achievement production largely deemphasizes the importance of the reflection problem in the educational context on the grounds that it is often irrelevant or insoluble.[3] The reflection problem is irrelevant if peer achievement spillovers derive only through peer ability, which can be proxied using lagged measures of peer achievement. However, I argue that treating peer effects as deriving solely through predetermined peer characteristics is inconsistent with much of the underlying theory, empirical and ethnographical evidence for peer effects and that contemporaneous peer achievement is likely to be important. Consequently, the reflection problem applies to the achievement context.

Returning to the issue of the insolubility of the reflection problem, consideration of the theoretical model also helps to highlight potential solutions. When individuals are treated as utility-maximizing agents, the achievement production function can be recast as an achievement best response function. Because students choose effort as a best response to their peers, this in turn presents the possibility of an exclusion restriction that would shift an individual's effort, and thus their achievement, independently of peers and separately identify the endogenous from the exogenous effects. This intuition cannot be extended to the peer ability case, simply because peer ability is predetermined. Furthermore, the model clarifies the properties of a valid exclusion restriction. When peer achievement is included among the peer effects, the exclusion restriction must enter through the utility function rather than the production function directly. It is not sufficient that a characteristic affects i's achievement but not his peers' achievement directly because this characteristic would still enter into the achievement of i's peers as a proxy for the unobservable peer effect.

[3]See Hanushek et al. (2003), Ammermueller and Pischke (2006), among others.

The question remains whether these insights are relevant for policy. While Manski (1993) emphasizes the importance of addressing the reflection problem for understanding social multiplier effects of policies, it may be that recovering reduced form estimates of the *social effect*, i.e., the combination of endogenous and exogenous effects, is sufficient for determining the effects of regrouping students. This is true in the linear-in-means context, in fact, when students are being regrouped based on observables. The literature recognizes this as another reason why the reflection problem may be of secondary importance in many policy applications.[4]

However, I highlight at least two reasons for caution in applying reduced form estimates to determining optimal groupings. First, while estimates of the reduced form are sufficient in the linear-in-means context, I show that this is sensitive to the assumption of homogeneous peer spillovers and random assignment across teachers. Second, while under the common parameters assumption the reduced form can be applied when regrouping is based on observables, i.e., the peer characteristics that form the social effects estimated in the model, more caution should be exercised when regrouping is based on unobservables, i.e., something that is observed to the policy maker and not the econometrician. For instance, separating race from "ability" effects may be important for policies such as determining the effect of shifting from integration based on race to race-blind policies that integrate based on measures of ability and income, and here the question whether "ability" is observed to the econometrician becomes important.

2 Interpreting Contextual Peer Effects

In other branches of the social interactions literature, the potential importance of endogenous effects for determining behavior is relatively self evident. Consider for instance the decision of a teenager to smoke or drink alcohol. Few would argue that the tendency toward such behaviors is unaffected by peer pressure, i.e., by having peers that engage in these behaviors. Yet, the role endogenous effects play in achievement is less clear, and this has led to considerable confusion regarding peer spillovers in the achievement production context. Annual standardized exams are often the outcome of interest, and, in the absence of cheating, are not a group effort. Thus, peer achievement per sé may not affect a student's achievement, a

[4]See, for instance, Ammermueller and Pischke (2006) for discussion.

point also recognized by Hanushek et al. (2003) and others.

Yet, despite this observation, some measure of peer achievement is often included among the potential peer effects in the achievement production function and is frequently even the input of interest. Why is this? In reality, peer achievement may signal something about peers that affects achievement production. There are several theories underlying the inclusion of peer achievement as an input to production. I distinguish between two–one that treats peer achievement as capturing unobserved characteristics of the student (such as ability) and another as capturing an unobserved action or behavior of the student (such as effort). The key distinction is in the timing. In a setting where we observe repeated observations of achievement on annual standardized exams, ability can be thought of as characteristic that is determined prior to the start of the academic year, whereas effort is determined during the academic year through interactions with peers, teachers and other influences.

The ability model effectively treats students as passive inputs to the production process, while the effort model explicitly recognizes the potential for students to respond to contemporaneous inputs, such as teachers and peers. Thus, while the literature generally lumps together both ability and effort-type spillovers as "endogenous" peer effects, the peer ability spillover may be more correctly characterized as an *unobserved exogenous effect*. In other words, at any given period t the peer ability spillover would not be endogenous if ability were observed, but is endogenous only because we use peer achievement to proxy for it. I will restrict the use of the term *endogenous* peer effects in this paper to those arising from contemporaneous peer spillovers.

To put some structure on the problem, let i index the individual student, t the academic year, g the peer group. For simplicity I assume that peer groups do not vary over time. Let Y_{igt} denote achievement on a standardized exam, which is taken at the end of each academic year. Observed individual characteristics, which often include parental education, race, sex, and some measure of income are denote X_{it}. K_{gt} captures observed classroom inputs are, such as teacher experience or expenditure, and μ_{gt} unobserved inputs, such as unobserved teacher quality, can affect achievement. The literature generally takes as a starting point an achievement production function with peer spillovers of the following form:

$$Y_{igt} = X_{it}\gamma_1 + \bar{X}_{gt}\gamma_2 + K_{gt}\gamma_3 + \gamma_4\bar{Y}_{gt-k} + \mu_{gt} + \xi_{igt}, \tag{2.1}$$

where peer spillovers derive both through mean peer characteristics $\bar{X}_{gt}$ (exogenous effects) and mean peer achievement $\bar{Y}_{gt-k}$, $k \geq 0$ ("endogenous" effect). This is taken to be the structural function in statistical models of peer effects. I will term it the statistical model, to distinguish it from the structural production functions described in the next sections that explicitly recognize the source of peer achievement spillovers as deriving through one or both of the unobservable channels.

In this section, I focus particularly on the implications of treating the "endogenous" peer effect as arising through unobservables for the interpretation of the exogenous peer effects, γ_2. Generally the intuition is that if a characteristic is beneficial to achievement (i.e., $\gamma_1 > 0$), the effect of having more peers with that characteristic should also be positive, or at least non-negative. However, estimates in the literature do not always bear out this intuition.[5]. For instance, Hanushek et al. (2003) find that having more peers receiving free/reduced price lunch is positively correlated with achievement, conditional on twice-lagged peer achievement. Cooley (2006) finds that having peers with better-educated parents is negatively correlated with achievement controlling for contemporaneous peer achievement.

I begin by explicitly modeling the ability spillover case, treating students as passive inputs to the achievement process. While I believe that this model misses the potentially more important behavioral effects of peers, this provides the clearest exposition of the implications for interpreting exogenous peer effects in the context where peer achievement is proxying for an unobservable. I then turn to the peer effort case in Section 2.2. While the implications for exogenous peer effects are similar, the contrast is important for identification.

Throughout the section I make the assumption that the econometrician chooses to use achievement to proxy for the unobservable. This contrasts with the question of whether peer achievement satisfies the conditions needed to make it a *good* proxy. The fact that it is not a good proxy in fact underlies some of the difficult identification problems and the "nonintuitive" interpretation of contextual effects described below. Thus, the discussion of identification in Section 3 can also be interpreted as addressing the problems associated with peer achievement not meeting the criteria of a good proxy.

[5]For instance, see Table 1

2.1 Ability Model

To simplify notation, I suppress peer group and time subscripts and focus on a particular peer group, such as a class, in a particular time period. Let A_i denote ability. I assume that the unobserved effect of peers is deriving through their ability ($\bar{A}$), so that achievement production is

$$Y_i = X_i\alpha_1 + \bar{X}\alpha_2 + \tilde{K}\alpha_3 + \alpha_4 A_i + \alpha_5 \bar{A} + \epsilon_i, \tag{2.2}$$

where $\tilde{K} \equiv (K, \mu)$. The econometrician wants to know the effect of peer ability. I maintain the assumption that achievement is monotonically increasing in ability, so that peer achievement can be used as a proxy. Solving for average peer ability as a function of peer achievement and other inputs and substituting for peer ability in equation (2.2) yields

$$\begin{aligned} Y_i &= X_i\alpha_1 + \bar{X}\left(\alpha_2 - \frac{(\alpha_1+\alpha_2)\alpha_5}{\alpha_4+\alpha_5}\right) + \tilde{K}\alpha_3\left(1 - \frac{\alpha_5}{\alpha_4+\alpha_5}\right) + \alpha_4 A_i + \frac{\alpha_5}{\alpha_4+\alpha_5}\bar{Y} + \left(\epsilon_i - \frac{\alpha_5}{\alpha_4+\alpha_5}\bar{\epsilon}\right) \\ &\equiv X_i\tilde{\gamma}_1 + \bar{X}\tilde{\gamma}_2 + \tilde{K}\tilde{\gamma}_3 + \tilde{\gamma}_4\bar{Y} + \tilde{\gamma}_5 A_i + \epsilon_i - \tilde{\gamma}_4\bar{\epsilon}. \end{aligned} \tag{2.3}$$

The question of interest is how to interpret the exogenous effects parameter, $\tilde{\gamma}_2$. First, note that as long as $\alpha_5 \geq 0$, the endogenous effects parameter $\tilde{\gamma}_4 \in [0, 1)$.

Suppose further that there is no direct effect of peer characteristics on achievement, so that $\alpha_2 = 0$ and $\tilde{\gamma}_2 = -\alpha_1\tilde{\gamma}_4$. Given $\alpha_1 \neq 0$ and $\alpha_5 > 0$, the exogenous peer effect still enters the achievement production function but takes on the opposite sign of the individual effect α_1.

If there are direct spillovers from peer characteristics in achievement production the sign of the contextual effect in the statistical model is ambiguous because of the countervailing influences of the indirect effect of peer characteristics as proxying for unobserved peer ability and the direct effect of peer characteristics in achievement production. Intuitively, this suggests that the stronger the spillovers from peer ability, the stronger the direct effect of the individual characteristic, and the weaker the direct effect of peer characteristics, the more likely $\tilde{\gamma}_2$ is to take a "counterintuitive" sign.

Assuming that α_1 and α_2 take the same sign, as often suggested by intuition, $\tilde{\gamma}_2$ takes the same sign as the direct effect of peer characteristics (α_2) if $\frac{\alpha_2}{\alpha_1} \geq \frac{\tilde{\gamma}_4}{1-\tilde{\gamma}_4}$. If in addition own characteristics have a larger direct effect than peer characteristics, i.e., $\alpha_1 > \alpha_2$, then $1 > \frac{\alpha_2}{\alpha_1} \geq \frac{\tilde{\gamma}_4}{1-\tilde{\gamma}_4}$ and a necessary condition is that $\tilde{\gamma}_4 < .5$, which is true for most estimates of peer "ability" spillovers.

A particular example of an exogenous effect may help clarify. Suppose we consider the effect of parental education. The literature generally supports the finding that students with better-educated parents perform better in school on average. This could follow if for instance better-educated parents value education more and are more able or more willing to spend time teaching their child outside of the classroom, helping on homework assignments, and facilitating other activities conducive to achievement.

But, what is the rationale for an effect of the parental education of peers on achievement? At one level, parents are not in the classroom and therefore cannot directly affect achievement production. Yet, peer parental education could potentially affect the productivity of the teacher, for instance, if better-educated parents spend more time monitoring and this has positive spillovers for the classroom. However, conditional on a given set of teacher inputs, peer parental education may have less of a direct effect on the productivity of the classroom. Therefore, this suggests a setting where conditioning on teacher productivity we could find a negative effect of peer parental education on achievement, simply because the indirect effect of peer parental education as a proxy for peer ability dominates the direct effect.

2.2 Effort Model

I now turn to a model that explicitly treats students as decision-makers and provides a starting point for interpreting the statistical model of the production function in equation (2.1) as arising from a model of peer effort spillovers. To obtain an achievement equation that is linear-in-means, both in peer achievement and characteristics, I impose fairly strict functional form assumptions on both utility and achievement production.[6]

The model also serves to solidify the intuition regarding the potential importance of contemporaneous peer spillovers. In particular, the literature provides support for at least two ways peer effort-type spillovers may affect achievement. First, peer effort can impose direct externalities on student achievement, i.e., disruptive peers detract from the learning environment with negative consequences to achievement. Second, peers may play a role

[6] Cooley (2006) describes the derivation of an achievement best response from an equilibrium model of effort that is similar in nature to the statistical model described in 2.3, but of a more general functional form.

in setting norms of conduct that may provide social pressures against or in favor of high achievement. While either is sufficient to motivate endogenous peer effects, I permit both types of spillovers in the model described below.

Let e_i denote effort, which is intended to capture a wide range of behaviors that are more or less conducive to achievement, such as working hard in class, asking more or less productive questions, misbehaving or distracting classmates, or cooperating in group work. I assume that the utility for student i can be written as

$$U_i = \beta_{1i} Y_i - \frac{\beta_2}{2} e_i^2 + \beta_3 e_i \bar{e},$$

where $\beta_{1i} \geq 0$, $\beta_2 \geq 0$, $\beta_3 \geq 0$.Students derive utility from achievement, and there is a cost to effort. However, the cost of effort is diminishing in the average effort of peers, what Brock and Durlauf (2001*a*) term the *proportional spillovers* case. Intuitively, this picks up the notion that there are costs to deviating from the norm (the average behavior of the peer group). To provide a familiar example in the context of achievement, the cost of working hard in a class of non-hard-working peers is likely to be much larger because the student risks standing out as a "nerd" or "teacher's pet."[7] Allowing the marginal utility of achievement to vary across individuals permits variation in utility-maximizing effort.[8] Note that this type of heterogeneity is new to the literature on social interactions, where generally heterogeneity is driven by errors in the residual. This will prove to be an important generalization for identification and is discussed more below.

Previous studies have also found support for a direct effect of peer effort on achievement. For instance, Lazear (2001)'s model of peer influence predicts that the disruptive behavior of a student imposes negative externalities on other students in the classroom. Both Figlio (2003) and Kinsler (2006) present empirical evidence that disruptive peers negatively affect achievement. Equally plausible is the potential positive externality of being grouped with more engaged students. Thus, the achievement realized by student i is a function of his own

[7] For instance, see Bishop et al. (2003). In a competitive environment, students may value achievement mostly as it relates to their peers' achievement, producing a similar style peer effect in determining optimal behavior.

[8] Note that as it is the marginal rate of substitution that matters, restricting β_2 to be homogeneous is not without loss of generality. However, in a model of endogenous reference group formation it seems likely that β_3 might also be individual-specific, i.e., where individuals place more weight on the actions of peers more "like" themselves. While this has interesting implications, it is beyond the scope of the present paper.

effort, the effort of his peers, and other exogenous inputs, i.e.,

$$Y_i = X_i\alpha_1 + \bar{X}\alpha_2 + \tilde{K}\alpha_3 + \alpha_4 A_i + \alpha_5\bar{A} + \alpha_6 e_i + \alpha_7\bar{e} + \epsilon_i. \tag{2.4}$$

Given that students simultaneously choose effort to maximize expected utility, a student i's best response to any given level of average peer effort can be described as

$$e_i^{BR} = \frac{\beta_{1i}\alpha_6}{\beta_2} + \frac{\beta_3}{\beta_2}\bar{e}.$$

Utility-maximizing effort is a function of the marginal utility of effort relative to the cost and is increasing in the average effort of peers as a result of the conformity effect.[9] In previous work, Cooley (2006), I show the informational assumptions and other conditions needed for a Nash equilibrium to exist in a more general setting. In the present context, the equilibrium described below is consistent with various types of informational assumptions given the additive separability in the residual and other classroom inputs.

Assuming $\alpha_6 > 0$ so that achievement is monotonically increasing in effort, the effort best response maps into an achievement best response, which is observable to the econometrician. Solving for average peer effort as a function of achievement using the production function in (2.4), we have

$$\bar{e} = \frac{1}{\alpha_6 + \alpha_7}\left(\bar{Y} - \bar{X}(\alpha_1 + \alpha_2) - \tilde{K}\alpha_3 - \bar{A}(\alpha_4 + \alpha_5) - \bar{\epsilon}\right).$$

We can then rewrite the effort best response as a function of average peer achievement, i.e.,

$$e_i^{BR} = \frac{\beta_{1i}\alpha_6}{\beta_2} + \frac{\beta_3}{\beta_2}\frac{1}{\alpha_6 + \alpha_7}\left(\bar{Y} - \bar{X}(\alpha_1 + \alpha_2) - \tilde{K}\alpha_3 - \bar{A}(\alpha_4 + \alpha_5) - \bar{\epsilon}\right).$$

Plugging i's best response into the achievement function, we have the achievement best

[9] Note that allowing for effort and peer effort complementarities in the achievement production function would suggest that the best response is increasing in average peer effort even in the absence of the conformity effect.

response as

$$
\begin{aligned}
Y_i^{BR} =& \frac{\alpha_6^2 \beta_{1i}}{\beta_2} + X_i\alpha_1 + \bar{X}\left(\alpha_2 - \frac{\beta_3\alpha_6 + \beta_2\alpha_7}{\beta_2(\alpha_6+\alpha_7)}(\alpha_1+\alpha_2)\right) + \tilde{K}\alpha_3\left(1 - \frac{\beta_3\alpha_6+\beta_2\alpha_7}{\beta_2(\alpha_6+\alpha_7)}\right) + A_i\alpha_4 \\
&+ \bar{A}\left(\alpha_5 - \frac{\beta_3\alpha_6+\beta_2\alpha_7}{\beta_2(\alpha_6+\alpha_7)}(\alpha_4+\alpha_5)\right) + \frac{\beta_3\alpha_6+\beta_2\alpha_7}{\beta_2(\alpha_6+\alpha_7)}\bar{Y} + \epsilon_i - \frac{\beta_3\alpha_6+\beta_2\alpha_7}{\beta_2(\alpha_6+\alpha_7)}\bar{\epsilon}, \\
\equiv& \gamma_{0i} + X_i\gamma_1 + \bar{X}\gamma_2 + \tilde{K}\gamma_3 + \gamma_4\bar{Y} + \gamma_5 A_i + \gamma_6\bar{A} + \epsilon_i - \gamma_4\bar{\epsilon}.
\end{aligned}
$$

Given that the achievement best response is linear-in-parameters, it can be shown that a unique Nash equilibrium exists to this game, $(Y_1^*, ..., Y_N^*)$. The observed equilibrium achievement as a function of individual and peer characteristics can then be written as

$$
Y_i^* = \gamma_{0i} + X_i\gamma_1 + \bar{X}\gamma_2 + \tilde{K}\gamma_3 + \gamma_4\bar{Y}^* + \gamma_5 A_i + \gamma_6\bar{A} + \epsilon_i - \gamma_4\bar{\epsilon}. \tag{2.5}
$$

Before turning to the interpretation of the exogenous peer effect, it is first useful to describe the properties of γ_4. For expositional purposes, I assume $\alpha_7 \geq 0$ so that effort and peer effort are weakly complementary inputs to achievement production. This is consistent with the idea that harder working peers create a better learning environment. While the effort best response is increasing in peer effort given the assumption of a conformity effect ($\beta_3 > 0$), $\alpha_7 \geq 0$ ensures that the achievement best response is also increasing in average peer achievement, i.e., $\gamma_4 \geq 0$. Furthermore, to ensure that the average peer achievement is positive, requires that $\gamma_4 < 1$, which holds if $\beta_2 > \beta_3$ or the direct cost of effort exceeds the conformity effect. Note, as an aside, that this is enough to ensure that shared inputs $\tilde{K}$ enter the semi-structural achievement production function with the same sign as their marginal product, α_3.

Therefore, the interpretation of γ_2 follows similarly to the ability case above. The larger the individual effect and endogenous effect, and the smaller the exogenous peer effect, the more likely that γ_2 take the opposite sign of γ_1. However, note that while the exogenous effects parameter takes a similar form in both models, there is no reason to expect the magnitude of the bias away from capturing the direct effect of peer characteristics to be the same. In fact, if contemporaneous spillovers are larger than peer ability spillovers as might be expected, the estimates of a peer effort model are further away from capturing the true direct effect of peer characteristics.

Note that as it is written, the equilibrium effort does not vary explicitly by a student's observable characteristics. Suppose we allow the marginal utility of achievement to depend on observable characteristics of the individual student and his peers that affect production directly and also potentially other observables Z_i, i.e., $\beta_{1i} = \beta_{10} + X_i\beta_{11} + \bar{X}\beta_{12} + Z_i\beta_{13}$.

In this case equation (2.3) becomes

$$Y_i^* = \gamma_0 + X_i\gamma_1 + \bar{X}\gamma_2 + \tilde{K}\gamma_3 + \gamma_4\bar{Y}^* + Z_i\gamma_7 + \gamma_5 A_i + \gamma_6\bar{A} + \epsilon_i - \gamma_4\bar{\epsilon}, \tag{2.6}$$

where $\gamma_0 = \frac{\alpha_6^2\beta_{10}}{\beta_2}$, and $\gamma_2 = \alpha_2(1-\gamma_4) - \gamma_4\alpha_1 + \frac{\alpha_6^2}{\beta_2}\beta_{12}$, and $\gamma_7 \equiv \frac{\alpha_6^2\beta_{13}}{\beta_2}$. The addition of peer characteristics in the utility function makes the spillovers from peer characteristics more likely to take the expected sign. Furthermore, allowing for preferences over achievement and effort to vary by observable characteristics highlights an alternative channel through which individual characteristics, such as parental education, may affect achievement production, i.e., through student motivation rather than as a direct input to production.

2.3 Discussion

The above arguments are based on the assumption that peer achievement itself does not matter for production. In a context where we are considering only direct externalities of peers on achievement production this seems justified, i.e., the externalities are coming through behaviors rather than achievement. In contrast, when students are treated as optimizing agents and choose behaviors based on peers, as suggested in the tracking and acting white literature, there may be a direct role for prior peer achievement in determining effort. For instance, if students are placed with peers who are higher performing, as they observe through knowledge of prior achievement, they could choose to work harder to maintain a certain status in the class. However, as long as this is accompanied by direct externalities from peer effort on production or responses to peer effort, the treatment of peer achievement as a proxy for unobservable peer characteristics remains important.

Table 1 provides some suggestive evidence in support of these observations from estimates in the literature. Most notably, as mentioned above, the exogenous effect parameter appears more likely to take on the “counterintuitive” sign when the peer achievement effect is larger in magnitude.

3 Identification

In the previous section, I discuss how the parameters in the statistical production function with peer spillovers might be interpreted in the context of a model that makes explicit the intuition that peer achievement spillovers derive through unobserved channels. Yet, a problem that is central in the literature is how to identify the contextual and endogenous peer effects. In this section, I emphasize how the model helps inform identification, connecting to the previous section by emphasizing what is being identified in terms of the underlying theoretical model. While the ability- and effort-model of peer achievement spillovers have similar implications for interpreting contextual effects, I show that the implications for identification are quite different. Because selection into peer groups has received considerable attention elsewhere and is tangential to the present discussion, I maintain the assumption of random assignment to peer groups.

A natural question is whether identifying the parameters of the statistical model of peer spillovers is useful, given the difficulties of interpretation highlighted in the previous section. Estimates of the endogenous effect parameter and the magnitude of the individual effect are useful in informing the extent to which the direct contextual effect is likely to be biased downward. In fact, in some settings estimates of the social effect may be sufficient for optimal grouping. Thus, I begin by considering the assumptions needed to identify the reduced form parameters. In Section 4, I attempt to link these results by describing the contexts in which these parameters can be applied to address questions of optimal grouping.

I begin by writing down the statistical equation for achievement with peer spillovers, equation (2.3), explicitly accounting for variation across peer groups (classrooms) g and school years t and restricting the marginal utility of achievement, β_{1i}, to be the same across students. Recall that $\tilde{K}_{gt} \equiv (K_{gt}, \mu_{gt})$, where μ_{gt} is unobservable to the econometrician. Achievement can then be expressed as a function of peer achievement and other inputs, i.e.,

$$Y_{igt} = \gamma_0 + X_i\gamma_1 + \bar{X}_{gt}\gamma_2 + K_{gt}\gamma_3 + \gamma_4\bar{Y}_{gt} + \mu_{gt} + \xi_{igt}, \tag{3.1}$$

where $\xi_{igt} \equiv \gamma_5 A_{it} + \gamma_6 \bar{A}_{gt} + \epsilon_{igt} - \gamma_4 \bar{\epsilon}_{gt}$. I allow A_{it} to vary over time to permit the interpretation of ability as human capital accumulation rather than just innate ability.

Solving for average peer achievement and substituting back into equation (3.1), we have the reduced form equation for peer achievement, i.e.,

$$Y_{igt} = \pi_0 + X_i\pi_1 + \bar{X}_{gt}\pi_2 + K_{gt}\pi_3 + \zeta_{igt}, \tag{3.2}$$

where

$$\begin{aligned}
\pi_0 &= \frac{\gamma_0}{1-\gamma_4}, \\
\pi_1 &= \gamma_1, \\
\pi_2 &= \frac{\gamma_2 + \gamma_1\gamma_4}{1-\gamma_4} \\
\pi_3 &= \frac{\gamma_3}{1-\gamma_4}, \\
\zeta_{igt} &= \gamma_5 A_i + \frac{\gamma_4\gamma_5 + \gamma_6}{1-\gamma_4}\bar{A}_{gt} + \frac{1}{1-\gamma_4}\mu_{gt} + \epsilon_{igt}.
\end{aligned}$$

As originally discussed by Manski (1993), existence of a social effect, defined as $\gamma_2 \neq 0$ and/or $\gamma_4 \neq 0$, can be determined given it is possible to recover consistent estimates of π_2. Interestingly, given the model discussed in Section 2.2, the social effect actually is the direct effect of the peer characteristics, i.e., $\pi_2 = \alpha_2$. Intuitively, this follows because a student's choice of effort is only affected by his peers' effort, i.e., neither his own or his peers' characteristics matter. This contrasts with models that begin by assuming that peer achievement belongs in the production function. Because peer achievement proxies for an unobserved peer effort spillover, there is no indirect effect of peer characteristics deriving through peer achievement, the second component of π_2 above. This result does not hold up when the marginal utility of achievement is a function of observable characteristics, as discussed below.

Much of the literature focuses on the identification of the existence of a social effect, rather than distinguishing between endogenous and exogenous effects. For least-squares to obtain consistent estimates of $\overrightarrow{\pi}$, a necessary condition is that $E(\zeta_{igt}|X_i, \bar{X}_{gt}, K_{gt}) = 0$. This breaks down into the following conditions:

(A1.) $E(A_i|X_i, \bar{X}_{gt}, K_{gt}) = 0$,

(A2.) $E(\bar{A}_{gt}|X_i, \bar{X}_{gt}, K_{gt}) = 0$,

(A3.) $E(\mu_{gt}|X_i, \bar{X}_{gt}, K_{gt}) = 0$.

As this has received considerable attention elsewhere in the literature,[10] I briefly summarize some of the concerns and solutions below. However, the focus in this paper is to highlight how the model helps inform identification of the endogenous effect parameter, which follows further below.

Under random assignment, the primary concern with (A1) and (A2) is that the observed characteristics and unobserved ability of a student are likely to be correlated. Whether this is a problem is likely to be context-specific. For instance, if policy makers are looking for ways of optimally assigning students to classrooms, it may be that differentiating between effects deriving through ability, which they do not observe, and observable peer characteristics is not important. In other words, the question would simply be one of interpretation. For instance, we may find that having peers with better-educated parents raises achievement, but we remain agnostic about whether this relationship is picking up the fact that these peers are likely to be more "able" (through intergenerational human capital transfer-type stories) or whether the fact that the parents are better-educated in itself is helping in some way.

(A3) requires that the unobserved group productivity, the correlated effect, is independent of observed inputs. Under random assignment, this requires that resource inputs do not change systematically with peer characteristics. If μ_{gt} captures teacher quality or other unobserved teacher inputs, this suggests that teachers do not vary inputs according to the characteristics of their class. On the one hand, this assumption seems unreasonable because it severely limits the role of teachers. On the other, there may be considerable stickiness in the teacher's inputs so that the teacher does not respond to relatively minor variations in class characteristics from year to year. This raises the deeper question of the distinction between peer and teacher effect. For instance, if a teacher given a class of low-ability students teaches a curriculum targeted to their ability and the same teacher given a mixed-ability class teaches a curriculum targeted to the middle, in the broadest definition this could be construed as a peer effect. Yet, this contrasts with the type of peer spillover that may be generated by a teacher who sticks to the same inputs each year but this input becomes more effective with say a homogeneous rather than a homogeneous class, an agglomeration-type effect.

I now turn to the question of separately identifying γ_2 and γ_4 in equation (3.1). It is easily shown that the endogenous and exogenous peer spillovers are not separately identified from

[10] See Brock and Durlauf (2001*b*) for overview.

the reduced form parameters, as follows from the original result in Manski (1993). Intuitively, nonidentification results because, as expanded by Brock and Durlauf (2001*b*), all individual effects are also contextual effects in the model. While this follows by assumptions on the statistical model in Manski (1993), the present model predicts this structure, namely that there is no individual effect whose associated contextual effect does not enter the statistical model of achievement, i.e.,

Proposition 1. *Given the theoretical model underlying equation (3.1) is of the form described in Section 2.2 and $\beta_{1i} = \beta_1$ for all i, no exclusion restriction exists and γ_2 and γ_4 are not identified.*

Proof. The number of reduced form parameters is less than the number of parameters in the statistical model, i.e., $1 + 2dim(X) + dim(K) < 2 + 2dim(X) + dim(K)$. □

One implication of this result is to emphasize how the use of peer achievement to proxy for peer effort eliminates any potential exclusion restrictions deriving through direct inputs to the achievement production function. For instance, even if we thought that a student's parental education affects his achievement but not the parental education of his peers in the structural production function, equation (2.4), this would not be a viable exclusion restriction in the context where we condition on peer achievement as a proxy for peer effort. However, relaxing the assumptions of the model to permit effort to vary by observable characteristics of the student opens the possibility of identifying the endogenous effect parameter.

Returning to the simple modification where the marginal utility of achievement varies by observable student characteristics, making explicit variation across time and peer groups in equation (2.6), we have

$$Y_{igt} = \gamma_0 + X_i\gamma_1 + \bar{X}_{gt}\gamma_2 + K_{gt}\gamma_3 + \gamma_4\bar{Y}_{gt} + Z_{it}\gamma_7 + \gamma_5 A_i + \gamma_6\bar{A}_{gt} + \mu_{gt} + \xi_{igt}. \tag{3.3}$$

Note that only the individual's own utility parameters enter into the achievement best response function because i is making a best response to any level of achievement (effort) of his peers. As a result, Z_{it} enters the achievement best response function, the statistical model we are estimating, only as an individual effect without a contextual counterpart, providing the exclusion restriction needed for identification, i.e., it acts as a utility shifter. To be explicit, the reduced form equation is now

$$Y_{igt} = \pi_0 + X_i\pi_1 + \bar{X}_{gt}\pi_2 + K_{gt}\pi_3 + Z_{it}\pi_4 + \bar{Z}_{gt}\pi_5 + \tilde{\zeta}_{igt}.$$

Given that $dim(Z_{it}) \geq 1$, we have that there are at least as many reduced form as parameters of the statistical model, i.e., $1 + 2d_1 + d_2 + 2d_3 \geq 2 + 2d_1 + d_3$. Effectively, the average peer utility-shifter $\bar{Z}_{gt}$ provides a potential exclusion restriction. However, it must also be independent of unobservables, most notably the unobserved group effect μ_{gt}. Sufficient conditions for identification are formally stated in the following proposition, as a counterpart to the nonidentification result in the previous section.

Proposition 2. *Suppose (i.) there exists a utility shifter, Z_{it}, such that $dim(Z_{it}) \geq 1$ and Z_{it} does not affect achievement production directly (i.e., it is not an input to equation (2.4)) and $\beta_{13} \neq 0$, $\alpha_7 \neq 0$, (ii.) $E(\mu_{gt} + \xi_{igt}|X_i, \bar{X}_{gt}, K_{gt}, Z_{it}, \bar{Z}_{gt}) = E(\mu_{gt} + \xi_{igt}|X_i, \bar{X}_{gt}, K_{gt}, Z_{it}) = 0$. Then, the parameters of the statistical model in equation (3.3) γ_2 and γ_4 are identified.*

Conditions (i) and (ii) ensure that $\bar{Z}_{gt}$ provides a valid instrument. Condition (i.) requires that Z_{it} does indeed affect achievement but through utility-maximizing effort and that $\bar{Z}_{gt}$ would then shift peer effort (achievement). Condition (ii.) is comparable to the Assumption (A1)-(A2) above with the additional covariates Z_{it} and $\bar{Z}_{gt}$, the familiar condition that the instrument not be correlated with the residual in the structural equation.

To consider some examples of potential exclusion restrictions, a policy or program that affects the incentives of some students in the peer group but not others may be useful. Cooley (2006) offers one example–the introduction of student accountability standards, which threaten students with retention if they do not perform above a certain level. Relying on the idea that only "low-achievers" suffer the threat of this policy, the instrument is then the percentage of peers held accountable. Also, it may be possible to exploit supplemental educational services provided to students under NCLB, under a similar idea that they motivate lower-achieving students to work harder producing spillover to their classroom peers. Another potential exclusion restriction is a family-level characteristic that affects choice of effort. For this to work, however, it cannot affect achievement directly. One example might be the presence of a high-achieving sibling.

One particularly useful feature of this extension is that it provides a more explicit role for parents or other home inputs. Complementarities between effort and characteristics are likely to enter the best response in other ways. For instance, complementarities between a student's effort and his own or his peers' characteristics in achievement production would produce similar results to the above case. Utility-maximizing effort would then be increasing in own or peer characteristics because the marginal product of effort is increasing in these inputs. Furthermore, an argument could be made that marginal utility is either increasing or decreasing in achievement. If marginal utility is increasing in achievement, then students with "better" X_i would want to exert relatively more effort, which would produce similar results to the above framework. Alternatively, if the marginal utility of achievement is diminishing, this would lead to the opposite effect. Intuitively, the key to identification is then that the exclusion restriction must not enter the production function directly and thus in the present model enters through preferences.

Throughout, I have maintained assumptions that generate a linear-in-means model of achievement production with peer spillovers, which as Brock and Durlauf (2001*b*) emphasize, provides a sort of worst-case scenario for identification. Cooley (2006) shows how the identifying assumptions can be extended to a more general framework using an effort-type model with complementarities, though the central insight for the exclusion restriction is effectively the same as described above.

Finally, it is worth noting that the reduced form parameter does not equal the direct contextual effect in this more general setting. In terms of structural parameters,

$$\pi_2 = \alpha_2 + \frac{\alpha_6^2(\beta_{12} + \beta_{11}\gamma_4)}{\beta_2(1-\gamma_4)},$$

recovers the social effect, which is equivalent to the direct effect in this case plus an additional term, which captures the effect of peer characteristics deriving through the effects on utility-maximizing effort choices and the social multiplier induced by having more peers exerting higher or lower effort. This is more closely related to the characterization of the reduced form parameters used by Manski (1993), and others.

3.1 Ability Model

Now, I briefly contrast the implications for identification in the ability model. The statistical model described in equation (2.3) in this case can be written as

$$Y_{igt} = \tilde{\gamma}_0 + X_i\tilde{\gamma}_1 + \bar{X}_{gt}\tilde{\gamma}_2 + K_{gt}\tilde{\gamma}_3 + \tilde{\gamma}_4\bar{Y}_{gt} + \mu_{gt} + \tilde{\xi}_{igt},$$

where $\tilde{\xi}_{igt} \equiv \tilde{\gamma}_5 A_i + \epsilon_{igt} - \tilde{\gamma}_4\bar{\epsilon}_{gt}$. A nonidentification result similar to Proposition 1 applies to the ability model, i.e.,

Proposition 3. *Given that peer ability is determined prior to year t and the underlying model is of the peer ability type described in Section 2.1, no exclusion restriction exists by which to identify the peer achievement spillover.*

However, in contrast to the peer effort model, relaxing assumptions to permit heterogeneity in preferences does not suggest potential channels for identification. Intuitively this follows because ability is predetermined and thus peer ability cannot be shifted once the peer group is set. Any direct effect on peer achievement also enters indirectly as a proxy for peer ability in the statistical model, so no exclusion restriction exists. The identification problem is thus fundamentally different across the two models.

The frequent approach taken in the literature is to eliminate the apparent simultaneity problem by substituting some measure of lagged peer achievement into the achievement production function,[11] i.e.,

$$Y_{igt} = \tilde{\gamma}_0 + X_i\tilde{\gamma}_1 + \bar{X}_{gt}\tilde{\gamma}_2 + K_{gt}\tilde{\gamma}_3 + \tilde{\gamma}_4\bar{Y}_{gt-1} + \mu_{gt} + \tilde{\xi}_{igt}, \tag{3.4}$$

and we can thus treat peer achievement as a predetermined variable. Then under the assumption that $E(\tilde{\xi}_{igt} + \mu_{gt}|X_i, \bar{X}_{gt}, \bar{Y}_{gt-1}, K_{gt}) = 0$, the parameters $\vec{\tilde{\gamma}}$ are identified. This can then be treated equivalently to identifying the parameters of the reduced form equation (3.2) above, with the following conditions:

(A1'.) $E(A_i|X_i, \bar{X}_{gt}, K_{gt}, \bar{Y}_{gt-1}) = 0$,

(A2'.) $E(\mu_{gt}|X_i, \bar{X}_{gt}, K_{gt}, \bar{Y}_{gt-1}) = 0$.

[11] See Hanushek et al. (2003) for discussion.

(A2') differs from (A3) in the effort case, in that the unobserved productivity μ_{gt} must be mean independent of average peer achievement in the previous period. This imposes the restriction that the unobserved productivity of the student's classroom, such as teacher quality, cannot be correlated over time. This condition may be unrealistic in most cases. For instance, teacher quality may be similar over time or "better" students may consistently get more attention from teachers.

This type of solution cannot be applied to a context where peer effort spillovers matter, if effort is influenced by inputs from the current academic year. In fact, though the empirical literature has focused primarily on ability-type spillovers, there is considerable evidence to suggest that effort-type spillovers may be important. Intuition suggests that teachers in particular may have an important role for motivating students and thus determining effort.[12] Furthermore, Stinebrickner and Stinebrickner (n.d.) show that ability is not generally a good predictor of effort, that effort varies considerably over time and has a large effect on achievement. That peer effort may affect achievement is supported by evidence of peer behavioral spillovers and conformity effects as discussed in Section 2.2. Lagged peer achievement would not capture any of these types of peer spillovers.

If we take as given that peer effort matters, an alternative way to think about the problem, which is relevant to policy applications, is to interpret the peer ability specification as a reduced form of the peer effort model. Effectively, this means treating $\bar{Y}_{gt-1}$ as a peer characteristic, and equation (3.4) as the reduced form of the achievement best response described in equation (3.1). Again, this would suggest that in estimating $\tilde{\gamma}$ in equation (3.4), the researcher is acknowledging that the lagged peer achievement is another contextual effect and it is not necessary to distinguish the contextual effect from the endogenous effect of peer effort. This becomes relevant in the next section, when the policy maker want to know the effect of ability grouping and lagged measures of peer achievement are the best measure of that ability.

It is worth emphasizing that using lagged peer achievement as a proxy does not remove the problem with the interpretation of exogenous effects as discussed in Section 2. In fact, if peer groups do not change over time, and the X_{it} are time-invariant, as is the case with most characteristics available in administrative data, the interpretation of exogenous effect parameter is the same conditioning on lagged or contemporaneous achievement. However,

[12]For instance, Roderick and Engel (2001) discuss the important role of teachers in determining how students responded high stakes testing.

if peer groups change over time, the indirect effect of conditioning on lagged achievement is weaker to the extent the average peer characteristics then vary over time. In other words, peer ability is estimated as a function of lagged peer achievement, peer characteristics and the lagged exogenous peer effects, among other inputs. Thus, even if there is a lot of variation in the lagged exogenous peer effect, created by variation in peer groups, the negative indirect effect deriving through the effect of peers' own characteristics on their lagged achievement will still bias estimates of the exogenous peer effect toward 0.

4 Implications for Policy: Optimal Grouping

While the previous sections developed intuition for the theoretical interpretations of the coefficients being estimated in the statistical models of peer achievement and the potential importance of contemporaneous spillovers for outcomes, the question remains how to link these observations to policy questions. For instance, an argument for ignoring the reflection problem in the achievement context is that reduced form estimates of the social effect are often sufficient for policy.

One well-known distinction between exogenous and endogenous spillovers is that only the latter generates social multiplier effects, as discussed by Manski (1993), Brock and Durlauf (2001*b*), Glaeser et al. (2003), Graham (2004), among others. Given that such social multipliers exist, the implications for policy can be quite different. To illustrate, consider the No Child Left Behind Act, which encourages schools to shift emphasis toward traditionally disadvantaged students, and therefore also potentially away from traditionally advantaged students. If endogenous effects are present, the improvements in the achievement of disadvantaged students will spillover to advantaged students. Furthermore, the gains from the increased resources to disadvantaged students will also be multiplied through peer spillover effects. Therefore, failure to take into account the social multiplier effects could severely misstate the effect of these types of policies. Furthermore, since these social multipliers are present only with spillovers of the effort-, rather than the ability-type, estimating contemporaneous peer achievement effects is important in this context.

However, one context where the reduced form estimates may be useful is for optimal grouping-type questions, which investigate the ceteris paribus effect of altering peer group composition on achievement. Often the questions of interest center on observable characteristics. Does tracking students by prior achievement improve performance? If increased school choice leads to exit of the children with better-educated or higher-income parents, does this hurt the students left behind? Does racial integration improve the performance of black students? Given that peer groups are altered based on observable measures, this begs the question whether the distinction between endogenous and exogenous spillovers matters in practice. In other words, the reduced form estimates precisely the relationship of interest, the correlation of observable peer characteristics with outcomes. The reason *why* these characteristics matter may then be of secondary importance for answering the types of optimal grouping questions described above. In this section, I explore the extent to which this is the case.

Because the literature on racial composition effects is extensive and of continued concern to policy makers, I center the discussion in this context. I maintain the assumption that consistent estimates of the reduced form parameters are available, i.e., the π's in equation (3.2) or the $\tilde{\gamma}$'s in equation (3.4), when it is interpreted as the reduced form of equation (3.1). To be clear, by *consistent* I mean that the parameters are capturing peer effects rather than some unobserved correlated effect. In equation (3.2), peer racial composition could be proxying for a whole host of characteristics such as ability, income, or effort, and should be interpreted accordingly.

4.1 Grouping on Observable Contextual Effects

Given the difficulties inherent in separating effort or ability spillovers from contextual peer effects, it is useful to begin by considering whether reduced-form estimates of the relationship between observable peer composition measures (other than lagged peer achievement) and achievement, as described by equation (3.2), are sufficient for determining optimal groupings in the linear-in-means model.

While π_2 describes the effect of marginal changes in peer composition, to really understand the implications of larger scale reallocation requires taking into account that by necessity a decrease in the percentage black in one classroom means an increase in another (assuming class size constraints and fixed student population). The larger policy question might then be how moving from a segregated to an integrated setting affects achievement. A well-known property of the linear-in-means model is that average achievement is not changed by altering peer composition–the gains to one group from any reallocation are perfectly offset by the losses to another. However, reallocation under the linear-in-means model does have equity implications, affecting the average achievement of subgroups. Most particularly, recent federal initiatives, namely the No Child Left Behind Act, have focused attention on raising achievement of racial minorities. Thus, for illustrative purposes I take the average achievement of black students as the outcome of interest.

Note that in the reallocation context, by necessity students are not only reassigned to peers but also to teachers. The ideal experiment to isolate the effect of regrouping might be to fix teacher quality at some value for all classrooms and then quantify the effect of reassigning students to peers. This would allow us to isolate the racial composition effect from the unobserved correlated effects. To illustrate, consider an example with two classrooms, $g \in \{c, d\}$ and two students each. Initially the allocation g_0 is such that students $\{1, 2\}$ are in c and $\{3, 4\}$ in d. I write down a simplified version of the statistical model with contemporaneous spillovers, equation (2.5), ignoring observable classroom inputs and suppressing time subscripts, i.e.,

$$
\begin{aligned}
Y_{1c} &= X_1\gamma_1 + X_2\gamma_2 + Y_{2c}\gamma_4 + \mu_c + \xi_{1c}, \\
Y_{2c} &= X_2\gamma_1 + X_1\gamma_2 + Y_{1c}\gamma_4 + \mu_c + \xi_{2c}, \\
Y_{3d} &= X_3\gamma_1 + X_4\gamma_2 + Y_{4d}\gamma_4 + \mu_d + \xi_{3d}, \\
Y_{4d} &= X_4\gamma_1 + X_3\gamma_2 + Y_{3d}\gamma_4 + \mu_d + \xi_{4d}.
\end{aligned}
$$

The characteristics X_i are understood to include a dummy variable for race. The associated reduced form is

$$
\begin{aligned}
Y_{1c} &= X_1\pi_1 + X_2\pi_2 + \pi_3\mu_c + \zeta_{1c}, \\
Y_{2c} &= X_2\pi_1 + X_1\pi_2 + \pi_3\mu_c + \zeta_{2c}, \\
Y_{3d} &= X_3\pi_1 + X_4\pi_2 + \pi_3\mu_d + \zeta_{3d}, \\
Y_{4d} &= X_4\pi_1 + X_3\pi_2 + \pi_3\mu_d + \zeta_{4d},
\end{aligned}
$$

where $\pi_1 \equiv \frac{\gamma_1+\gamma_4\gamma_2}{1-\gamma_4^2}$, $\pi_2 \equiv \frac{\gamma_2+\gamma_4\gamma_1}{1-\gamma_4^2}$, $\pi_3 = \frac{1+\gamma_4}{1-\gamma_4^2}$, $\zeta_{ik} \equiv \frac{\xi_{ik}+\gamma_4\xi_{jk}}{1-\gamma_4^2}$. Suppose we are able to obtain consistent estimates of the reduced form parameters π_1, π_2. Note that because there are no exclusion restriction, $\gamma_1, \gamma_2, \gamma_4$ are not identified. Suppose students $\{1, 2\}$ are black. We want to know how the average achievement of black students changes when we move from the observed homogeneous arrangement g_0 to an integrated grouping g_1. For instance if we group together students $\{1,3\}$ in c and $\{2,4\}$ in d, the associated reduced form would be

$$\begin{aligned} Y_{1c} &= X_1\pi_1 + X_3\pi_2 + \pi_3\mu_c + \zeta_{1c}, \\ Y_{2d} &= X_2\pi_1 + X_4\pi_2 + \pi_3\mu_d + \zeta_{2d}, \\ Y_{3c} &= X_3\pi_1 + X_1\pi_2 + \pi_3\mu_c + \zeta_{3c}, \\ Y_{4d} &= X_4\pi_1 + X_2\pi_2 + \pi_3\mu_d + \zeta_{4d}. \end{aligned}$$

The average change in achievement for black students is then

$$E(Y_1 + Y_2|g_1) - E(Y_1 + Y_2|g_0) = [(X_3 + X_4) - (X_1 + X_2)]\pi_2 + E(\pi_3(\mu_d - \mu_c)|\vec{X}),$$

where I have assumed that $E(\zeta_i|\vec{X}) = 0$. Under the assumption of random assignment of students to classrooms (and hence unobserved teacher quality μ), the reduced form estimates can be applied to predict the effect on average black achievement of moving from the extreme of a perfectly segregated setting to an integrated setting. This follows because of the common parameters assumption and random assignment, which effectively acts in expectation as if teacher quality were fixed.

However, random assignment is not the norm, and often researchers attempt to recover racial composition effects in a context where there is sorting of students and teachers to peer groups. A fundamental problem in identifying racial composition effects from longitudinal, observational data is that generally schools with higher concentrations of black students are also those that are worse in unobservable ways, violating the assumption that $E(\mu_{gt}|\bar{X}_{gt}) = 0$. Teacher quality in particular is notoriously difficult to measure, though both empirical and anecdotal evidence suggest that it matters.[13] The literature recognizes this problem. One method used to separately identify racial composition effects from unobserved correlated effects is to exploit some type of within-school variation.[14] Evidence in Clotfelter et al. (2006) suggesting that most teacher-student matching occurs between schools supports the use of school fixed effects to correct for this type of selection.

Under the assumption that teachers are matched to students, with predominantly white schools having relatively higher teacher quality on average than predominantly black schools,

[13]For instance, Clotfelter et al. (2006) find evidence that more highly qualified teachers tend to be matched with more affluent schools or schools with fewer minority students. See also Hoxby (2006) for overview.

[14]For instance, see Hoxby (2000), Ammermueller and Pischke (2006), Vigdor and Nechyba (forthcoming) and Hanushek et al. (2004).

the reduced form estimates can still be applied to predict the effect of integrating classes. Returning to the 4 student example to illustrate, we can recover $\mu_c^* \equiv \pi_3 E(\mu_c|X_1, X_2)$ and $\mu_d^* \equiv \pi_3 E(\mu_d|X_3, X_4)$ as the residual from the reduced form equations. This is sufficient for predicting the change in average achievement of black students under the regrouping, i.e., $\pi_3 E(\mu_d - \mu_c)|\vec{X}) = \mu_d^* - \mu_c^*$. Thus, under the common parameters assumption of the traditional linear-in-means model, reduced form estimates can be used to determine the effect of regrouping students even when unobserved group effects vary systematically with peer group composition.

However, both theory and empirical evidence suggest that the linear-in-means model may not provide a good approximation of peer effects in practice. In particular, evidence suggests that blacks and whites may respond differently to peers, with important implications for the usefulness of desegregation policies to narrow the racial achievement gap.[15] For instance, Hanushek et al. (2004) find that the percentage of black peers has a stronger negative effect on black students than on whites. Echenique and Fryer (2006) find that the degree of segregation within schools, measured by the cross-racial social interactions, for black students is highly nonlinear and increases with the percentage black of the school. This heterogeneity in the social effect could follow either as a result of heterogeneous responses to contextual effects or endogenous effects or both.

Furthermore, in determining the potential costs of academic tracking, it is often assumed that low-achievers benefit relatively more from being grouped with higher-achieving peers than high achievers. In this context, mixed-ability classes provide the highest average achievement. Extending to the question of desegregation, given that black students are more highly concentrated in the lower tails of the achievement distribution, integration could also raise average achievement. The literature generally supports the existence of these types of nonlinearities.[16] Thus, as Hoxby (2000) emphasizes, moving beyond linear-in-means models of peer effects is likely to be critical for developing interesting policy implications.

Returning to the previous example, consider the case where there are heterogeneous

[15]For instance, see Fordham and Ogbu (1986), Cooley (2006), Hanushek et al. (2004), Hoxby and Weingarth (2005), Fryer and Torelli (2005), among others.

[16]See, for instance, Cooley (2006), Ding and Lehrer (2006), Hanushek et al. (2003), Hoxby and Weingarth (2005) for evidence of nonlinearities. Ammermueller and Pischke (2006) is an exception.

responses to peer achievement across races, i.e., the statistical model is modified so that

$$
\begin{aligned}
Y_{1c} &= X_1\gamma_{1B} + X_2\gamma_{2B} + Y_{2c}\gamma_{4B} + \mu_c + \xi_{1c},\\
Y_{2c} &= X_2\gamma_{1B} + X_1\gamma_{2B} + Y_{1c}\gamma_{4B} + \mu_c + \xi_{2c},\\
Y_{3d} &= X_3\gamma_{1W} + X_4\gamma_{2W} + Y_{4d}\gamma_{4W} + \mu_d + \xi_{3d},\\
Y_{4d} &= X_4\gamma_{1W} + X_3\gamma_{2W} + Y_{3d}\gamma_{4W} + \mu_d + \xi_{4d},
\end{aligned}
$$

where W denotes white and B black. The associated reduced form equations are then

$$
\begin{aligned}
Y_{1c} &= X_1\pi_{1B} + X_2\pi_{2B} + \pi_{3B}\mu_c + \zeta_{1c},\\
Y_{2c} &= X_2\pi_{1B} + X_1\pi_{2B} + \pi_{3B}\mu_c + \zeta_{2c},\\
Y_{3d} &= X_3\pi_{1W} + X_4\pi_{2W} + \pi_{3W}\mu_d + \zeta_{3d},\\
Y_{4d} &= X_4\pi_{1W} + X_3\pi_{2W} + \pi_{3W}\mu_d + \zeta_{4d},
\end{aligned}
$$

where $\pi_{1r} \equiv \frac{\gamma_{1r}+\gamma_{4r}\gamma_{2r}}{1-\gamma_{4r}^2}$, $\pi_{2r} \equiv \frac{\gamma_{2r}+\gamma_{4r}\gamma_{1r}}{1-\gamma_{4r}^2}$, $\pi_{3r} = \frac{1+\gamma_{4r}}{1-\gamma_{4r}^2}$, and $r \in \{B, W\}$. If we were to reassign students to create heterogeneous classes as before the reduced form would be

$$
\begin{aligned}
Y_{1c} &= X_1\pi_{1BW} + X_3\pi_{2BW} + \pi_{3BW}\mu_c + \zeta_{1c},\\
Y_{2c} &= X_2\pi_{1BW} + X_4\pi_{2BW} + \pi_{3BW}\mu_d + \zeta_{2d},\\
Y_{3d} &= X_3\pi_{1WB} + X_1\pi_{2WB} + \pi_{3WB}\mu_c + \zeta_{3c},\\
Y_{4d} &= X_4\pi_{1WB} + X_2\pi_{2WB} + \pi_{3WB}\mu_d + \zeta_{4d},
\end{aligned}
$$

with $\pi_{1WB} \equiv \frac{\gamma_{1W}+\gamma_{4W}\gamma_{2B}}{1-\gamma_{4W}\gamma_{4B}}$, $\pi_{2WB} \equiv \frac{\gamma_{2W}+\gamma_{4W}\gamma_{1B}}{1-\gamma_{4W}\gamma_{4B}}$, $\pi_{3WB} = \frac{1+\gamma_{4W}}{1-\gamma_{4W}\gamma_{4B}}$. This suggests at least two problems in using the reduced form to estimate effects of regrouping. First, given heterogeneous responses to peers, it may not be possible to infer the effect of regrouping from the observed groupings. Given a sufficiently rich support, however, and a sufficiently flexible estimator, this problem is likely to be of secondary importance. For instance, in this example if we were to observe other racially mixed classrooms, then we could plausibly estimate $\pi_{1WB}, \pi_{2WB}, \pi_{1BW}, \pi_{2BW}$.

However, the second, and perhaps more difficult problem, is how to deal with unobserved teacher quality that may be correlated with the racial composition of the classroom. In this case, knowing the endogenous effect is critical for isolating the effect of teacher quality from peers. The intuition is simple. Suppose we fix teacher quality at some value $\bar{\mu}$, which is the average teacher quality in the district. In the perfectly integrated system, black students would receive higher teacher quality on average and white students lower. Without estimates of the social multiplier effect, it would not be possible to separate an effect of racial integration from a teacher effect. This suggests that the applicability of reduced form estimates to determining the effects of altering peer groups is particularly fragile to the assumption of homogeneous responses to peer achievement.

To provide a concrete example, suppose we want to estimate the effect of creating racially integrated classrooms using administrative data. We assume that students are randomly assigned within schools. Thus, controlling for school fixed effects obtains consistent estimates of the reduced form parameters. However, given that racial composition is correlated with school quality, these estimates can only be used to estimate the effect of regrouping within schools, not across school. Given that much racial segregation occurs between schools in a district, this severely limits our ability to determine the effect of integration policies that operate at the district level.

Conditioning on Achievement In comparison to historical studies that estimate desegregation effects from comparisons of outcomes before and after integration, one benefit of peer effects in achievement style studies is in isolating the relative importance of these different channels of influence, such as prior achievement, racial and income composition. Given the recent Supreme Court ruling that it is not constitutional in many circumstances to use race as a basis for assigning students to schools, a number of districts have attempted to maintain some degree of integration by assigning students based on other observables, such as free/reduced price lunch status and prior achievement,[17] A question of interest is then whether these race-blind integration policies can substitute for racial integration in terms of achievement effects.

In this context, the relationship of interest may be the effect of racial composition conditional on lagged peer achievement, as in equation (3.4).[18] Recall that the interpretation of the exogenous effect parameter, $\tilde{\gamma}_2$, is complicated by the fact that it is picking up the direct

[17] See Hoxby and Weingarth (2005) for example using Wake County's reassignment plan.

[18] Note that in the case where equation (3.4) is interpreted as the reduced form version of the effort model, the relation of $\tilde{\gamma}_2$ to the structural parameters is still as described in equation (2.3), i.e., the social effect is picking up the direct spillover less the indirect spillover deriving through the peer achievement proxy for ability. Again, this result follows because characteristics do not enter effort choices.

effect less the indirect effect deriving through peer ability, as discussed in Section 2.[19]

Yet, from the discussion so far it is unclear whether α_2 or $\tilde{\gamma}_2$ is more relevant for policy purposes. Given that the linear-in-means model is correct and assignment is being made on the lagged values of the observable standardized achievement measures, the coefficient on racial composition in equation (3.4) can be correctly interpreted as describing whether racial composition matters after controlling for average peer achievement. However, this is not true if policy makers integrate based on some ability that is unobserved to the econometrician, i.e., IQ scores, classroom performance, or parental assertiveness. While standardized achievement scores may be a valid proxy for ability, the coefficient on percentage nonwhite cannot be interpreted as whether or not racial composition matters conditional on ability. The distinction is subtle, but could nonetheless be important for deriving policy implications.

Returning to the two student, two class example to illustrate, suppose that we estimate a simplified version of the ability model in equation (3.4)

$$\begin{aligned}
Y_{1ct} &= X_1\tilde{\gamma}_1 + X_2\tilde{\gamma}_2 + Y_{2t-1}\tilde{\gamma}_4 + \mu_{ct} + \xi_{1ct},\\
Y_{2ct} &= X_2\tilde{\gamma}_1 + X_1\tilde{\gamma}_2 + Y_{1t-1}\tilde{\gamma}_4 + \mu_{ct} + \xi_{2ct},\\
Y_{3dt} &= X_3\tilde{\gamma}_1 + X_4\tilde{\gamma}_2 + Y_{4t-1}\tilde{\gamma}_4 + \mu_{dt} + \xi_{3dt},\\
Y_{4dt} &= X_4\tilde{\gamma}_1 + X_3\tilde{\gamma}_2 + Y_{3t-1}\tilde{\gamma}_4 + \mu_{dt} + \xi_{4dt},
\end{aligned}$$

where for the moment we remain agnostic as to whether this is the reduced form or the statistical model. Suppose in reality we believe that it is ability that matters rather than prior peer achievement, i.e.,

$$\begin{aligned}
Y_{1ct} &= X_1\alpha_1 + X_2\alpha_2 + A_1\alpha_4 + A_2\alpha_5 + \mu_{ct} + \xi_{1ct},\\
Y_{2ct} &= X_2\alpha_1 + X_1\alpha_2 + A_2\alpha_4 + A_1\alpha_5 + \mu_{ct} + \xi_{2ct},\\
Y_{3dt} &= X_3\alpha_1 + X_4\alpha_2 + A_3\alpha_4 + A_4\alpha_5 + \mu_{dt} + \xi_{3dt},\\
Y_{4dt} &= X_4\alpha_1 + X_3\alpha_2 + A_4\alpha_4 + A_3\alpha_5 + \mu_{dt} + \xi_{4dt}.
\end{aligned}$$

While this is written as a simplified version of the structural production function in the ability model, equation (2.2), it could also correspond to the reduced form of the effort model, in

[19]Note that in the simplest setting where effort is not a function of own or peer characteristics, the direct effect can be recovered from the estimates of $\tilde{\gamma}$, the semi-structural parameters, i.e., $\alpha_2 = (\tilde{\gamma}_2+\tilde{\gamma}_4\tilde{\gamma}_1)/(1-\tilde{\gamma}_4)$. Note, however, that when individual or peer characteristics help determine utility-maximizing effort, this result no longer holds.

which case $\alpha_1 = \pi_1$ and $\alpha_2 = \pi_2$. A contrast however is that in the present form reallocation of unobserved teacher quality is not a concern because there are no social multipliers in the ability model. Thus, if we treated this as a reduced form of the effort model, the same intuition from the previous section would apply.

If it were possible to recover consistent estimates of the α's, the change in achievement for black students would be

$$E(Y_1+Y_2|g_1)-E(Y_1+Y_2|g_0) = [(X_3+X_4)-(X_1+X_2)]\alpha_2+\alpha_5[(A_3+A_4)-(A_1+A_2)]+E(\mu_{dt}-\mu_{ct}).$$

In contrast, using the model with prior achievement we have

$$E(Y_1+Y_2|g_1)-E(Y_1+Y_2|g_0) = [(X_3+X_4)-(X_1+X_2)]\tilde{\gamma}_2+\tilde{\gamma}_4[(Y_{3t-1}+Y_{4t-1})-(Y_{1t-1}+Y_{2t-1})]+E(\mu_{dt}-\mu_{ct}).$$

There are at least two contrasts to emphasize. First, this example corresponds to moving from homogeneous to heterogeneous grouping on prior achievement or "ability" if we assume that the black students in the example have lower prior achievement and similarly other measures of human capital than whites. However, it is possible that regrouping based on prior achievement versus other measures of ability may lead to different groupings, which would lead to different outcomes than what is assumed here. Second, grouping based on ability that is unobserved to the econometrician versus prior achievement suggests both different estimates of the exogenous effects conditional on prior achievement and also different contrasts in terms of the change in peer "ability." In sum, the implication is that policy makers should use extreme caution in developing implications for regrouping based on observed "ability" to regrouping based on unobserved "ability."

5 Conclusion

In this paper, I attempt to clarify the rationale for endogenous peer effects, i.e., the inclusion of peer achievement in the achievement production function. I take as an underlying premise that peer achievement per se does not matter in achievement production, but rather serves as a proxy for an unobserved quality of the peer group, as generally argued in the literature. I contrast two types of peer spillovers that peer achievement could capture–unobserved effort and ability. The important distinction is that only the former is truly endogenous, the latter being predetermined. The contrast is of practical importance because it suggests that using lagged measures of peer achievement, an approach generally taken in the literature, fails to capture important behavioral spillovers of peers.

In the paper I highlight three areas where the literature on peer effects in educational achievement may be misguided. The first is the tendency to ignore the reflection problem, minimizing the importance of simultaneity concerns for identification of peer spillovers. Lagged measures of peer achievement are generally preferred because they are less likely to be correlated with unobserved group effects, such as teacher quality, whereas the simultaneity concerns associated with including contemporaneous peer achievement are thought to be insoluble. I argue rather that theory suggests natural exclusion restrictions that permit the identification of endogenous peer effects that are not apparent in the ability-based framework that is commonly assumed.

The second is that more careful consideration should be given to the interpretation of estimates of the spillovers from peer characteristics. Using peer achievement to proxy for unobserved peer "quality" suggests that peer characteristics may appear to be correlated with achievement even if they do not directly affect achievement, but only indirectly as a proxy for peer quality. Furthermore the indirect proxy channel works in opposition to the direct externality that is commonly assumed, suggesting that the intuition that a student should be positively affected by peers with characteristics conducive to achievement may not always bear out in estimates.

Third, I show that reduced form estimates of the social effect of peers are generally not sufficient for determining the effects of regrouping students on the achievement of different subgroups. Therefore, determining the causal effect of peers, i.e., separating endogenous from exogenous peer effects, is central to developing viable policy implications of large scale reallocations of students.

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Table 1: Recent Studies on Peer Achievement and Composition.

Studies	Context	Source /Specification	Peer Achievement	Sign /Magnitude	Peer Characteristics	Sign /Magnitude
Hanushek et al. (2003)	Public Schools Texas, US	Table 2, Column 3 Student Fixed Effects	Average Math Score T-2	0.15	Proportion Eligible for Reduced Price Lunch	0.1
Vigdor and Nechyba (2004)	Public Schools N. Carolina, US	Table 2, Columns 3 and 6, No Student Fixed Effects	Average Math Score T-2	0.0741	% Black in Classroom	-0.1327
Betts and Zau (2004)	Public Schools, San Diego, US	Table 3, Column 1, Student Fixed Effects	Average Math Score T-1	1.9295	% Black at School Level	Not Reported
Hanushek et al. (2004)	Public Schools Texas, US	Table 1, Column 7 Student Fixed Effects	Average Math Score T-2	0.01	% Black in Grade	-0.25
Henry and Rickman (2007)	Preschool Georgia, US	Table 4, Column 1	Cognitive Skills (WJ-AP)	0.39	% Black in Classroom	0.12
Gibbons and Telhaj (2005)	Public Schools, England	Table 5, Column 5	Average Math Score T-3	0.218	% White in School-Grade	-0.004
Cooley (2006)	Public Schools, N. Carolina	Table 3, Col. 4	Average Reading Score T	.48	% nonwhite in classroom	.10