Comparing Procedures and Concepts in Math Learning

Dissertation Abstract, Shanta Hattikudur

The focus of this dissertation is to investigate how comparison of concepts and procedures affects gains in procedural and conceptual knowledge. The relationship between procedural skills and conceptual understanding is one of the oldest concerns in the field of mathematical psychology (Resnick & Ford, 1981). Not only are procedural and conceptual knowledge important, but a particular subgroup of conceptual understanding—an integrated understanding of procedures and concepts—is essential to a complete understanding of mathematics. If the goal of instruction is to develop meaningful knowledge that can be transferred to new and unfamiliar problems, educators should create classroom instruction directed at promoting integrated thinking.

Perhaps one way to help students learn both concepts and procedures (and possibly integrate their knowledge of both) is through comparison. Research has shown that comparing multiple examples supports learning better than studying one example alone or two examples separately (e.g., Gentner & Namy, 2004; Gick & Holyoak, 1983; Hattikudur & Alibali, 2010; Oakes & Ribar, 2005; Rittle-Johnson & Star, 2007). In the past couple of decades, studies have suggested using comparison as a tool for mathematics classroom instruction, in which students actively compare, reflect on, and discuss multiple problems or solution methods (Ball, 1993; Fraivillig, Murphy,; Hufferd-Ackles, Fuson, & Sherin, 2004; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005). US teachers do make connections between examples or solution methods—unfortunately, these efforts often result in simple exposure to the multiple methods without discussion or active comparison of the methods (Ball, 1993; Chazan & Ball, 1999) or fail to provide students with the cognitive support needed to help them benefit from these connections (Richland, Zur, & Holyoak, 2007). With poor instructional support for comparison, the intended meaningful connections often go unnoticed (Bransford, Franks, Vye, & Sherwood, 1989; Chi, Feltovich, & Glaser, 1981; Gick & Holyoak, 1983). If comparison is going to be useful in the classroom, research needs to better inform teachers about how to use comparison effectively.

Although comparing examples of concepts or examples of procedures may support the learning of concepts and procedures, this dissertation sought to develop an even more effective comparison instruction—one guiding students to compare a concept with a procedure. The effects of learning through this new comparison instruction were tested using measures of students' procedural knowledge, conceptual knowledge, and a new measure of integrated knowledge of procedures and concepts in the domain of fraction division. Results indicated that comparison instruction in a laboratory and large group classroom setting fostered procedural, conceptual, and integrated knowledge more than sequential instruction. Students' attitudes towards math played a significant role in whether students benefited more from comparison or sequential instruction. In general, when learning concepts and procedures in fraction division, students with low math attitudes benefited more from comparison instruction and students with high math attitudes benefited more from sequential instruction. Students with high math attitudes did, however, gain an integrated understanding of the concept and procedure through comparison instruction more than sequential instruction. Thus, comparing concepts and procedures is an effective tool in instruction that could be used in the classroom in order to promote procedural, conceptual, and most importantly, an integrated learning of mathematics.